**AAI 672 Applied Game Theory and Evolutionary Algorithms**

**Midterm Exam Solutions– Fall 2022**

**Problem 1. (15 points)** Consider the strategic form game with the actions and payoffs given by the following matrix:

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | 6,-6 | -1, 1 |
| B | -5, 5 | 4, -4 |

1. Does the game have a pure strategy Nash equilibrium? If yes, identify it

No pure strategy NE

1. Identify the Pareto efficient strategy tuples

All tuples are Pareto efficient

1. Determine the mixed strategy Nash equilibrium for this game

Apply indifference principle:

Let p = prob that player 1 plays A

q = prob that player 2 plays C

For player 2: for player 2: -6p+5(1-p) = 1\*p+(-4)\*(1-p) 🡪 p = 9/16

For player 1: 6q-1(1-q) = -5q+4(1-q) 🡪 q = 5/16

**Problem 2. (10 points)** For the game above, assume now that the users do not know each other’s strategy of play and they would like to use fictitious play to learn the opponent’s strategies. Will the fictitious play algorithm converge? Explain.

Yes. The above game is a zero sum game, so the fictitious algorithm will converge to the mixed strategy NE determined above.

**Problem 3 (20 points).**

Assume that the classic prisoner dilemma game is played repeatedly 3 times, and the players know that the game is going to end after the third time period. The game is illustrated in the strategic form matrix:

Table

Description automatically generated with medium confidence

1. What is the NE for the one stage game?

For one stage game: NE is (D,D) with payoff (1,1)

1. What is the NE for the repeated game with 3 stages? Justify? Can a strategy that enforces cooperation be proposed for this case? Justify your answer.

Because the fact that the game will end after three stages in known by the players, this is a finite horizon game, which can be solved by applying backward induction. Because in the last stage of the game, players have incentives to defect, a cooperation strategy cannot be enforced. The NE can be determined by backward induction, and it will be (D,D).

1. Assume now that the game is planned to be played indefinitely, but it abruptly stops after 3 stages. Is this a different game than the one in (b)? Explain.

Since the game is designed to be played indefinitely and it ends unexpectedly after 3 stages, the game is an infinite horizon game, so it is a different game that the one described in b).

1. For the game in (c), can you propose a strategy that enforces cooperation between players for this case? For what value of the discount factor your proposed strategy can enforce cooperation?

The Grim strategy will enforce cooperation for a discount factor .

Grim strategy: start by cooperating and cooperate until the first user defects, then defect until the rest of the game.

An user has incentives to defect at one time if the gain from the one stage defecting outweighs the loss after defecting:

Assume Player 1 Defects at current time: payoff =

Cooperate: 3 +

Cooperate>= Defect 🡪 🡪

**Problem 4. (10 points)** Assume that, in a cellular system, all users aim to be received with the same power at the base station. To achieve this, users implement power control. The Base station (BS) expects that some of the users may have malicious behavior and will increase their powers for personal gain, inflicting more errors for other users. The BS will punish the jamming users by dropping their packets, such as to discourage a malicious behavior. Because of random channel characteristics, occasionally, with some probability p, a regular user will make a mistake in estimating the channel, which may lead to excess received power.

We model the BS-user game, in which the BS does not know if the user is malicious or not.

It is assumed that a malicious node will strategize on using higher power than allowed, so he will not get caught, i.e., he will used a higher power with probability q , while a regular user will accidently make mistakes with prob p (p much smaller than q).

1. Describe the game as a dynamic Bayesian game. Outline the main components of the game.

Players: BS – type known (only one type)

User – two types: malicious or regular

Strategies: punish, not punish

Malicious user strategy: use higher power with probability p (strategize), regular power

with 1-q

Regular user strategy : use higher power with probability q (by mistake), regular power

with 1-q

1. What type of equilibrium does this signaling game have (separating, pooling, semi-separating)?

Semi-separating equilibrium

**Problem 5. (15 points).** Construct your own coordination game, by selecting an appropriate game matrix (choose your own payoffs). Find the Nash equilibrium (or equilibria) for this game. What do you need to change in the payoff matrix to obtain a potential game? Find the potential function for your proposed game.

Each student will have his own game – they should construct it either as being a coordination game (simplest case), or coordination + dummy game. The potential function will have the values of utilities for the coordination game.

**Example:**

|  |  |  |
| --- | --- | --- |
|  | C | D |
| A | 3,3 | -1, -1 |
| B | 1, 1 | 5, 5 |

Potential function:

P(s1,s2)=